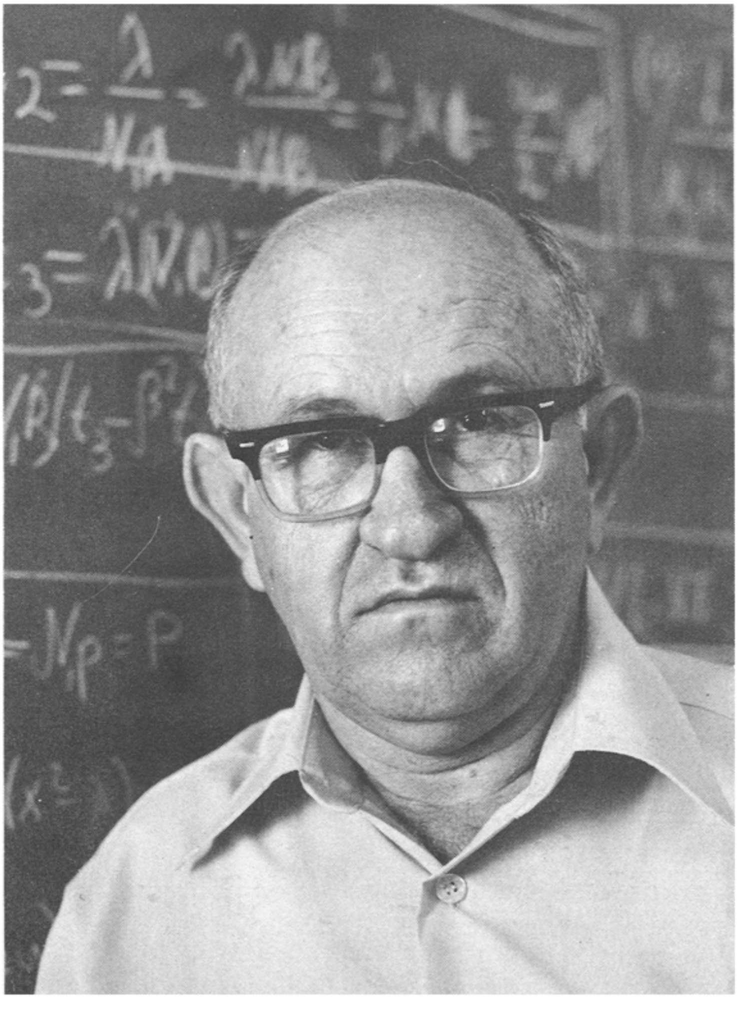

Shimshon Avraham Amitsur

1921–1994

ISRAEL JOURNAL OF MATHEMATICS **96** (1996), ix–xxvii

SHIMSHON AVRAHAM AMITSUR (1921 – 1994)

BY

AVINOAM MANN

Institute of Mathematics
The Hebrew University of Jerusalem, Jerusalem 91904, Israel

Shimshon Avraham Amitsur was born, under the name Shimshon Kaplan, on 26 August 1921, in Jerusalem, and passed away on 5 September 1994, in the same city. He is survived by his wife Sarah, his children Hanna, Michal, and Eli, and several grandchildren.

Several years after his birth his family moved to Tel Aviv, where he attended a commercial school. His talents were noticed by his teachers, and his principal, S. Maharshak, a well known writer of high school text books in mathematics, arranged a fund for his studies in the Hebrew University. These studies were interrupted by his army service both during World War II and during Israel's War of Independence. His letters to Levitzki show how he kept up his interest in mathematics even during this service. He received his M.Sc. degree in 1946, and his Ph.D. in 1950, both under the supervision of Professor Jacob Levitzki. Around that time he changed his name to the Hebrew one, Amitsur. Levitzki's report on the M.Sc. thesis is titled 'a report on the thesis of Shimshon Amitsur', but in the report itself Levitzki forgot the change of name and refers to Mr. Kaplan. Amitsur's appreciation of this great teacher of his was often witnessed by all his friends, and he was a very worthy successor to Levitzki in their jointly loved area, ring theory. Teacher and student received in 1953, for their joint work described below, the first Israel Prize for exact sciences, the most distinguished prize in Israel. Later Amitsur won several other prizes, and in 1990 was awarded the degree of Doctor of Philosophy (Honoris Causa) from Ben-Gurion University of the Negev.

The early death of Levitzki in 1956, aged 52, left Amitsur as the leading algebraist in Israel, a position he held indisputably till his death, even after his retirement in 1989. This retirement, marked by a two-week ring theory conference organized by the Hebrew University and Bar-Ilan University, did not stop his mathematical productivity in the least. His influence on Israeli mathematics was enormous, by no means restricted only to algebra, let alone ring theory. Witness his students: the first one worked in non-associative algebra, the second in group theory, another in category theory, and one of the last in Lie superalgebras. This diversity resulted from Amitsur's willingness to give his students free rein to choose their area. As one of these students, I can testify that, while he let me work in my preferred area, he was still very much interested in all that I did, monitoring it closely and giving valuable advice. He was always inquiring about my progress; it was very difficult to be slack while working under Shimshon. On the other hand, he would also tell me about his own progress, seeming, to my surprise, to regard me as an equal, and was always ready to share his knowledge. In all of this he set a remarkable personal example, no less instructive than direct mathematical advice.

Of course most of his students were in ring theory. Moreover, many ring theorists throughout the world, while not his students nominally, feel themselves so morally. For nearly fifty years he influenced the ring theory world greatly, through example, suggestion, and personal contact. The number of his posthumous articles, three of them in this volume alone, shows how he continued to be an active researcher till his last days.

Amitsur was one of the leaders of the Institute of Mathematics of the Hebrew University, contributing much to its development and seeing that it maintained its high level. He was also very active in the life of the university. Though he avoided offices such as dean etc., he was a member of many committees, and his voice was often heard in faculty meetings. He was a member of the Israel Academy of Science, in which he fulfilled many important duties, e.g. Head of the Section for Experimental Science. He was one of the founders and editors of the Israel Journal of Mathematics. He was also an editor of many other mathematical journals, and the mathematical editor of the Hebrew Encyclopedia (in that office he replaced the late A.A. Fraenkel).

Before passing to a description of his mathematical work, let me mention his great interest in mathematical education. For some time, he headed the Center for Teaching of Science of the University, and for many years was the motivating force in developing new programmes for the teaching of mathematics in Israeli high schools. The present day programmes are based on his ideas. He will be missed in this as in all his other activities.

The Mathematical Work of S.A. Amitsur

It will be impossible for me to describe in detail all, or even most, of Amitsur's more than one-hundred papers. The comments below are thus necessarily influenced by my personal inclinations, knowledge, and prejudices.

While Amitsur published mostly on ring theory, he occasionally digressed to other fields. Many of his readers will be surprised by the paper [18] in which Amitsur joined a pair of physicists in describing and analyzing an electric (not electronic!) analogue computer for the solution of systems of linear equations. That computer, a network of coils and condensers, was able to solve a system of nine equations in nine variables, up to an accuracy of one-thousandth, in one hour.

Better known is the set of papers [40, 45, 46, 48, 52, 62] in which Amitsur developed a symbolic approach to approximations of arithmetic functions, which leads to simplification in the elementary proofs of results such as the prime number theorem, the prime number theorem in arithmetic progressions, density results on the distribution of prime ideals, etc.

We now turn to Amitsur's ring theoretical papers. Since there are so many of them, we will discuss them under several headings, but we have to emphasize that any such division is somewhat artificial, and there are often connections between articles under different headings.

1. Polynomial identities

A ring R is said to *satisfy a polynomial identity* (a *PI-ring* for short) if there exists a non-zero non-commutative polynomial $\sum a_{(i)}x_{i_1}\cdots x_{i_k}$ (in other words, an element of the free associative algebra on a countable set of generators), which vanishes whenever elements from R are substituted for the variables x_i. An example is provided by the commutative rings, which satisfy the law $xy - yx = 0$. The notion of PI-rings was introduced by Max Dehn, with geometric motivation. Later it came to be regarded as a generalization of commutativity, the idea being that in such a ring it should be possible to develop a theory analogous to the successful theory of commutative rings. The importance of this class of rings was recognized by Jacob Levitzki, and he interested in it his gifted student Shimshon Amitsur. A finite-dimensional algebra over a field F, or more generally over a commutative ring, is a PI-ring. This applies in particular to the matrix algebra F_n, and the question arose: what are the identities satisfied by F_n, and in particular what are the *minimal* identities, i.e. those of minimal degree? Levitzki has shown that the degree of an identity for F_n is at least $2n$. In [9, 11] Amitsur and Levitzki not only showed that the minimal degree is exactly $2n$, but also identified the identities of this minimal degree. They are the so-called *standard identities*, i.e. the identity $S_{2n} := S_{2n}(x_1, \ldots, x_{2n}) := \sum sg(i)x_{i_1} \cdots x_{i_{2n}} = 0$, where the summation is over all permutations of the $2n$ variables, and the sign of each term is the sign of the corresponding permutation. Amitsur liked to describe the standard identity as the non-commutative determinant of size $2n \times 2n$, with all rows equal to the $2n$-tuple $(x_1, \ldots, x_{2n})$. The Amitsur–Levitzki Theorem, as this result is now known, became one of the cornerstones of PI-theory. The case of 2×2 matrices over a field of 2 elements, which is exceptional in this theorem, was discussed in [11].

In [19] it is shown how the standard identity can be modified to yield a two-variable identity which holds in F_n but not in matrix algebras of higher dimension. One corollary is a theorem of A.A. Albert, that each finite-dimensional division algebra can be generated by two elements. This is further refined in [25], to show that the two generators can be chosen to be conjugate.

In [17] Amitsur considered the set of all polynomial identities satisfied by a given PI-ring R. These form a subset I of the free associative algebra A, which is easily seen to be an ideal, and, moreover, to be invariant under all endomorphisms of A. Amitsur calls such ideals *T-ideals*. The ring A/I has the same identities as R, and indeed is the free ring (of countable rank) in the variety of all rings satisfying the identities of R. Amitsur calls this free ring a *universal PI-ring*, and studies its structure. He shows that its Jacobson radical is nil. Together with known results on (Jacobson) semi-simple PI-rings, it follows that each PI-ring satisfies an identity of the form $S_k^n = 0$, for some k and n (see also [68]). An interesting corollary of this study of the free ring is that any ring (not necessarily a PI one) is the homomorphic image of a subdirect sum of matrix algebras over $\mathbf{Z}$ (the ring of integers).

More properties of the universal PI-rings are developed in [27]. It is proved there, e.g., that the T-ideals of the free ring are primary.

The much later papers [93, 94] discuss the set of identities from another viewpoint. A PI-ring always satisfies a *homogeneous multilinear* identity, i.e. one that is linear in each of its arguments, and which is the sum of monomials each of which involves the same set of variables, say $x_1, \ldots, x_n$. The set A_n of all multilinear polynomials of degree n is a subspace of the free algebra, and the set of multilinear identities of degree n of R is a subspace I_n of A_n. Moreover, the symmetric group S_n acts on A_n and I_n. Amitsur's student Amitai Regev, in his Ph.D. thesis, considered the dimension of the difference space $A_n - I_n$, which he calls the nth *codimension* of R. Later he considered also the *cocharacters* of R, i.e. the characters of the representations of S_n on this difference space. The two papers mentioned above apply the representation theory of the symmetric groups to study these codimensions and cocharacters. This leads to very interesting results about the nature of the identities holding in R, but these are too technical to be described here.

We now come to papers investigating the structure of PI-rings. Here Amitsur was very much interested in embedding theorems. The matrix ring C_n over a commutative ring C is a PI-ring, indeed it satisfies the standard identity S_{2n}. Then each subring of this matrix ring is also PI. The converse does not hold: there are PI-rings which are not subrings of matrix rings over commutative rings. Yet PI-rings turn out to be closely related to subrings of matrix rings. In [14] it is proved that a PI-ring without nilpotent ideals is a subring of a matrix ring over a commutative ring. The assumption about nilpotent ideals is weakened in [26], where it is also proved that a PI-ring R without zero divisors is a so-called Ore domain. This is equivalent to R having a division ring of quotients, i.e. R is a subring of a division ring D, such that each element of D can be written as ab^{-1}, where a and b lie in R. This result was extended by Posner, who showed that a prime PI-ring has a semisimple ring of quotients. A different proof of that is given by Amitsur in [60]. In [66, 72] the problem of embedding in matrix rings is attacked "generically". It is shown that to each ring R and an integer n there corresponds a uniquely determined commutative ring S and a homomorphism $\phi: R \to S_n$, such that each homomorphism from R to a matrix ring C_n, where C is commutative, factors through ϕ. Some of Amitsur's students investigated the problem of finding S, for some specific rings R, but in general this interesting question seems to be still open.

An algebra over a field is called *affine*, if it is finitely generated as an algebra.

These algebras were always an important subject of study in PI-theory, e.g. the Jacobson–Levitzki–Kaplansky–Shirshov theorem states that an algebraic affine PI-algebra is finite dimensional. Amitsur [37] proved that the Jacobson radical J of an affine PI-algebra is nil. Building on that, A. Braun showed much later that J is actually nilpotent. Amitsur also obtained a non-commutative version of Hilbert's *Nullstellensatz*, studying zeroes of polynomials in matrix rings and division rings [37], and this was generalized by Amitsur and Procesi [57] to the theorem that in an affine PI-algebra all prime ideals are intersections of maximal ones. In [81, 89, 99, 106, 107] Amitsur and Small investigated prime ideals of prime affine algebras, and also generalized the JKLS theorem. These results led to the blossoming of algebraic geometry over affine PI-algebras.

In a completely different direction, Amitsur showed [63] that if a ring with involution (*) satisfies a (*)-identity, it is actually a PI-ring. This elegant result encompasses earlier ones by Herstein and others. This process, Amitsur taking an interesting result of somebody else and putting it in a broader context, was quite typical.

We should mention here also some results on nil PI-rings [12, 58], e.g. that the Baer series defining the lower nil radical of a PI-ring stops after two steps at most. I will mention by name only Amitsur's work on central polynomials, on applications of PI-theory to Azumaya algebras, on groups with representations of bounded degree (i.e. whose group ring is PI), and on rings with pivotal monomials, a generalization of PI-rings.

2. Rational identities

The problems in the foundations of geometry which led Dehn to define PI-rings, actually lead to identities which involve also inverses, and thus should be properly called *rational*, rather than polynomial, identities. Dehn avoided the study of these type of identities, and they remained virtually untouched till Amitsur's monumental paper [59]. In that paper Amitsur first of all defines precisely the notion of a rational identity, and studies division rings satisfying them. The main result is that if a rational identity holds in some division ring D, of dimension n over its center (n may be infinite), it also holds in all division rings of the same characteristic and of smaller dimension over their center. Thus the identities which hold in an infinite-dimensional division ring hold in all division algebras of the same characteristic. These identities are called *universal*, and are the ones that hold in all matrix algebras. Two division rings with infinite center satisfy the same rational identities if and only if they have the same characteristic and the same dimension over their center.

These very definitive results are obtained as the climax of a long chain of steps, in each of which it is shown that if an identity holds in D, it also holds in some other ring constructed from D. One of these rings is $D < x >$, the ring obtained by adjoining freely to D a non-commuting variable x. In order to deal with this free extension Amitsur developed first in [55] the theory of *generalized polynomial identities*, i.e. identities which are sums of monomials of the form $a_1 x_{i_1} a_2 \cdots x_{i_n} a_{n+1}$, with $a_i \in D$. That paper contains the main structure theorem for rings satisfying a GPI, characterizing the relevant primitive rings. This theory is noteworthy in its own right, but for Amitsur it was just one step in the theory of rational identities. In other steps Amitsur applies the theory of ultra-products. The application of this notion, which, though purely algebraic, occurred first in model theory and is still mostly used there, testifies to Amitsur's ability to learn and apply tools from all disciplines of mathematics when appropriate. I have on many occasions noted how when hearing of interesting new results Amitsur would always try to understand the methods that led to the new developments. (Speakers in the algebra seminar in Jerusalem (now called officially the Amitsur Algebra Seminar) can testify how closely he questioned them on their methods.) Often he was able to apply similar techniques later himself, with remarkable consequences.

Returning to rational identities, the above results have several corollaries. One is the construction, for a division ring D, of a ring which can be naturally described as the ring of rational functions over D. This yields a natural embedding of free rings in division rings. Another application is the complete solution of Dehn's original problem, determining the intersection theorems in projective geometry which are intermediate between Desargues's and Pappus's theorems. In view of the above theorems, any such intersection theorem is equivalent to a combination of statements about the characteristic and the dimension over the center of the coordinate division ring.

3. Division rings

In can be said that the general structure theory of rings aims to reduce this theory to the theory of division rings. Among these, progress has been achieved mainly for the ones that are finite dimensional over their center (they are known today as *division algebras*). If D is such a ring, with center Z, let F be a maximal subfield of D. Then $|D\colon F| = |F\colon Z|$. This common dimension, n say, is the *degree* of D. If F/Z happens to be a Galois extension, with Galois group G, then D is a so-called *crossed product* of F and G, and its structure is determined completely by F, G, and a certain cocycle. If G is cyclic, then D is termed *cyclic*. Wedderburn showed that division rings of degree 2 and 3 are cyclic, while Albert showed that this is no longer true for degree 4, but that rings of degree 4 are still crossed products. Possibly the most striking result in division ring theory to date is the Albert–Brauer–Hasse–Noether theorem, from the early thirties, stating that if D is finite dimensional over $\mathbf{Q}$ (the rationals; thus Z is an algebraic number field), then D is cyclic. The general case remained for forty years the central open problem in ring theory, and even partial results were scarce. In 1972 Amitsur published a counter example [71].*

The words "counter example" are perhaps not appropriate here. On hearing them one imagines some ingenious construction of a specific strange ring, with some ingenious argument, tailor-made for this example, showing that it has the desired properties. There is no doubt of the ingenuity of Amitsur's construction, but his example is far from esoteric. It is, rather, the most general division ring. To start with one considers d *generic* $n \times n$ matrices over $\mathbf{Q}$, i.e. matrices whose entries are independent (commuting) variables $x_{ij}^{(k)}$. These matrices generate a subring of the matrix ring over $\mathbf{Q}(x_{ij}^{(k)})$. This ring, by earlier results, has no zero divisors, and has a division ring of quotients D, which is of degree n. D is called the *generic* division ring. Amitsur employs the generic nature of this ring to show that if it is a crossed product, then so is any division ring of degree n and characteristic 0. The more surprising part of this result is that all these division rings are crossed products *with the same Galois group* as D. Now Amitsur analyses two special cases, of division rings over the p-adic field, and over a field of formal power series. In both cases it turns out that the division rings in question are indeed crossed products, and moreover, with a uniquely determined Galois group G. (In general it is possible for a ring to be a crossed product in several ways, with different groups.) In the first case this group, for an appropriate p, contains a cyclic subgroup of index 2 (Dirichlet's theorem on

* The example was first to be published in a volume of papers planned for Albert's 65th birthday, which did not appear due to Albert's death.

primes in arithmetic progressions is needed for the choice of p). In the second case G is a direct product of groups of prime order. Since G has order n, the two structures for G are incompatible if n is divisible by 8 or by the square of an odd prime. Thus for such values of n, the generic division algebra is not a crossed product.

It is noteworthy that the proofs of the above-mentioned properties of the generic division algebras rely heavily on the PI-theory that Amitsur was so instrumental in developing during the previous twenty years.

The method was modified later to apply also for other base fields, not necessarily $\mathbf{Q}$, provided n is divisible by the cube of some prime, and if the characteristic p is finite, provided also that p does not divide n. For p dividing n new ideas were required, which were supplied by Amitsur and Saltman in their fundamental study of abelian crossed products [83], and on this basis Saltman constructed non-crossed products also in this case. Thus there is a non-crossed product division algebra of degree n in all characteristics, provided n is divisible by some cube. What happens for n that does not satisfy this condition, say for $n = 5$, is still very much open.

In the series of papers [95, 96, 97, 102], Amitsur and Tignol consider the question how far the generic division algebra is from being a crossed product. More specifically, they obtain lower bounds for the dimensions of splitting fields for this algebra (splitting fields are defined in the next paragraph). In another direction, the Amitsur–Saltman theory is used by Amitsur–Rowen–Tignol [86, 87] to construct a division algebra with involution which is not a tensor product of quaternion algebras, thus answering another long-standing problem.

While the non-crossed product is Amitsur's most important contribution to division algebras, his interest in them dates from much earlier. A field C is a *splitting field* for a division algebra D if the tensor product of C and D is isomorphic to the matrix algebra C_n; e.g. the maximal subfields of D are splitting fields for it. In his thesis [5, 24; see also 22, 23, 31] Amitsur constructed for each D a certain transcendental extension of the center Z, which is a splitting field for D and, moreover, it determines the set of all splitting fields of D. Using this *generic* splitting field, it is shown that if A and B are two central simple algebras, then each splitting field for A splits also B, if and only if B is equal, in the Brauer group of Z, to a power of A. In particular A and B have the same splitting fields iff they generate the same subgroup of the Brauer group. The generic splitting field turns out to be of fundamental importance in the modern study of Severi–Brauer varieties.

Passing now to general division rings (i.e. not necessarily finite dimensional), another famous result of Amitsur is his determination of the finite subgroups of division rings [29]. These were known before for finite characteristics (they are cyclic then), but the characteristic 0 case is much harder, and to solve it Amitsur applies group theory on the one hand, and class field theory on the other. Another result, an immediate corollary of the theorem on rational identities, is that the multiplicative group of an infinite-dimensional division ring does not satisfy any non-trivial identity. In [84] Amitsur and Small obtain important results on polynomial rings in several variables over division rings. Amitsur's construction of cyclic extensions of division rings [22] should also be mentioned, and there is more.

4. Algebras over uncountable fields

This topic illustrates once again Amitsur's ability to absorb techniques from different fields. This time the key concept is the *spectrum* of an element of an algebra over a field. Again this purely algebraic concept was first defined, and widely used, in operator theory (there, the base field is either the real or the complex one). Amitsur applied it [30] to solve some of the outstanding problems in the theory of algebras, provided the base field F is big enough. Thus, if A is an algebra over F, and the cardinality of F exceeds the dimension of A over F, then the Jacobson radical of A is nil. If, moreover, A is a division ring, it is algebraic. A recent generalization by Amitsur and Small [110, in this volume] replaces the assumption that A is a division algebra by the one that all non-zero divisors are invertible. Returning to [30], if F is uncountable, and A is nil, or algebraic, then so are the matrix algebras over A, as well as the algebras obtained from A by extension of scalars. It is still unknown whether these propositions are true for countable base fields. It also follows that for algebras over an uncountable field there is a positive solution to the Köthe problem: the maximal nil ideal of A contains all one-sided nil ideals. From time to time Amitsur tried to attack the general Köthe problem, but it is still open.

5. The Amitsur complex

The Brauer group over a field F consists of all division algebras over F, with an operation induced by the tensor product. All algebras split by a given field K form a subgroup $\mathrm{Br}(K/F)$. If K/F is a Galois extension, with a Galois group G, then G acts on the mutiplicative group K^* of K, and this action gives rise to certain cohomology groups. One has then $\mathrm{Br}(K/F) \cong H^2(G, K^*)$. In [43] Amitsur constructs for each field extension K/F a complex which makes it possible to define cohomology groups also when K/F is not Galois. The above isomorphism holds then for these cohomology groups as well. The terms in the Amitsur complex are the groups of invertible elements of the tensor powers of K, and the homomorphisms are alternating sums of the $n+1$ natural embeddings of the nth tensor power in the $(n+1)$st one.

6. Further results

Of the other contributions of Amitsur, let us mention first his fundamental theorem [32] that the Jacobson radical of a polynomial ring $R[x]$ is of the form $N[x]$, where N is a nil ideal of R. Thus if R has no nil ideals, then $R[x]$ is semisimple. This often reduces problems on semiprime rings to ones on semisimple rings. In [38] a similar theorem is obtained for the radical of field extensions. This is applied in [44] to show, by an extremely short argument, that if F is a field of characteristic 0, transcendental over $\mathbf{Q}$, and G is any group, then the group ring $F[G]$ is semisimple. Amitsur also constructed an explicit ring of quotients of semiprime rings, and reworked Morita's theory in terms of matrix rings. Finally, his formulation of a general theory of radicals was also very influential on people interested in that area.

I am indebted to Joram Lindenstrauss, Louis Rowen, and Lance Small for their extremely helpful remarks on earlier versions of this article. In particular, the article incorporates many of Rowen's suggestions and formulations, often verbatim.

Other obituaries were written by Paul M. Cohn (Bulletin of the London Mathematical Society **28** (1996), 433–439), and by Louis Rowen (Algebra Colloquium **2** (1995), 1–2). A forthcoming edition of Amitsur's papers, to be published by the American Mathematical Society, will also include detailed summaries of his work. An overview of Amitsur's mathematics, written by N. Jacobson, is included in the proceedings of his retirement conference (*Ring Theory 1989, in honor of S.A. Amitsur*, edited by L. Rowen, The Weizmann Science Press of Israel, Jerusalem, 1989, pp. 1–11).

Students of S.A. Amitsur

1. Shmuel Schreiber (1963), *Loop rings.*
2. Avinoam Mann (1966), *The connection between the structure of subgroups and the structure of finite groups.*
3. Aaron Klein (1966), *Relations in categories.*
4. Avraham A. Klein (1966), *Embedding problems in ring-theory.*
5. Jonathan Samuel Golan (1971), *Characterization of rings over which certain modules are quasiprojective.*
6. Amitai Regev (1972), *Identities in the tensor product of algebras with identities.*
7. Uri Leron (1973), *Polynomial identities in rings.*
8. Eliyahu Rips (1975), *Equations in free groups.*
9. Amos Kovacs (1976), *Matrix rings over commutative rings and rings with identities.*
10. Aner Shalev (1989; joint supervision with A.Mann), *Dimension subgroups, nilpotency indices, and the number of generators of ideals in modular group rings.*
11. Avital Frumkin (1993), *Atypical representations of Lie superalgebras.*
12. Dalit Baum (1996; joint supervision with L. Rowen; A. Mann replaced Amitsur after his death), *Skew algebraic elements of simple Artinian rings.*

Publications of S.A. Amitsur

1. *Applications of the theory of linear differential equations I*, Riveon lematematica **1** (1946/7), 47–48 (in Hebrew; English summary).
2. *Applications of the theory of linear differential equations II*, Riveon lematematica **1** (1946/7), 79–82 (in Hebrew; English summary).
3. *On unique factorization in rings*, Riveon lematematica **2** (1948), 28–29 (in Hebrew; English summary).
4. *A generalization of a theorem on linear differential equations*, Bulletin of the American Mathematical Society **54** (1948), 937–941.
5. *Contributions to the theory of central simple algebras*, Summary of thesis submitted to the Hebrew University of Jerusalem, 1949, pp. 1–28.
6. *On a lemma of Kaplansky*, Riveon lematematica **3** (1949), 47–48, 52 (in Hebrew; English summary).

7. *La representation d'algebres centrales simples*, Compte rendus de l'Academie des Science, Paris **230** (1950), 902–904.

8. *Construction d'algebres centrales simples sur des corps de caracteristique zero*, Compte rendus de l'Academie des Science, Paris **230** (1950), 1026–1028.

9. *Minimal identities for algebras* (co-author J. Levitzki), Proceedings of the American Mathematical Society **1** (1950), 449–463.

10. *Finite differential polynomials*, Riveon lematematica **4** (1950), 1–8 (in Hebrew; English summary).

11. *Remarks on minimal identities for algebras* (co-author J. Levitzki), Proceedings of the American Mathematical Society **2** (1951), 320–327.

12. *Nil PI-rings*, Proceedings of the American Mathematical Society **2** (1951), 538–540.

13. *Semi-group rings*, Riveon lematematica **5** (1951), 5–9 (in Hebrew; English summary).

14. *An embedding of PI-rings*, Proceedings of the American Mathematical Society **3** (1952), 3–9.

15. *A general theory of radicals. I. Radicals in complete lattices*, American Journal of Mathematics **74** (1952), 774–786.

16. *The problem of Kurosh–Levitzki–Jacobson*, Riveon lematematica **5** (1952), 41–48 (in Hebrew; English summary).

17. *The identities of PI-rings*, Proceedings of the American Mathematical Society **4** (1953), 27–34.

18. *An electrical computer for the solution of linear simultaneous equations* (co-authors A. Many, U. Oppenheim), Review of Scientific Instruments **24** (1953), 112–116.

19. *Applications to a polynomial identity*, Riveon lematematica **7** (1954), 30–32 (in Hebrew; English summary).

20. *A general theory of radicals. II. Radicals in rings and bicategories*, American Journal of Mathematics **76** (1954), 100–125.

21. *A general theory of radicals. III. Applications*, American Journal of Mathematics **76** (1954), 126–136.

22. *Non-commutative cyclic fields*, Duke Mathematical Journal **21** (1954), 87–105.

23. *Differential polynomials and division algebras*, Annals of Mathematics **59** (1954), 245–278.

24. *Generic splitting fields of central simple algebras*, Annals of Mathematics **62** (1955), 8–43.

25. *Identities and generators of matrix rings*, Bulletin of the Research Council of Israel **5A** (1955), 5–10.

26. *On rings with identities*, Journal of the London Mathematical Society **30** (1955), 464–470.

27. *The T-ideals of the free rings*, Journal of the London Mathematical Society **30** (1955), 470–475.

28. *The radical ring generated by a single element*, Riveon lematematica **9** (1955), 41–44 (in Hebrew; English summary).

29. *Finite subgroups of division rings*, Transactions of the American Mathematical Society **80** (1955), 361–386.

30. *Algebras over infinite fields*, Proceedings of the American Mathematical Society **7** (1956), 35–48.

31. *Some results on central simple algebras*, Annals of Mathematics **63** (1956), 285–293.

32. *Radicals of polynomial rings*, Canadian Journal of Mathematics **8** (1956), 355–361.

33. *Invariant submodules of simple rings*, Proceedings of the American Mathematical Society **7** (1956), 987–989.

34. *Hermite rings and the equivalence of matrices*, Riveon lematematica **10** (1956), 41–45 (in Hebrew; English summary).

35. *The scientific work of Prof. Jakob Levitzki*, Riveon lematematica **11** (1957), 1–6 (in Hebrew; English summary).

36. *Derivations in simple rings*, Proceedings of the London Mathematical Society **7** (1957), 87–112.

37. *A generalization of Hilbert's Nullstellensatz*, Proceedings of the American Mathematical Society **8** (1957), 649–656.

38. *The radical of field extensions*, Bulletin of the Research Council of Israel **7F** (1957/1958), 1–10.

39. *Countably generated division algebras over non-denumerable fields*, Bulletin of the Research Council of Israel **7F** (1957/1958), 39.

40. *On arithmetic functions*, Journal d'Analyse Mathematique **5** (1956/1957), 273–314.

41. *Commutative linear differential operators*, Pacific Journal of Mathematics **8** (1958), 1–10.

42. *Rings with a pivotal monomial*, Proceedings of the American Mathematical Society **9** (1958), 635–642.

43. *Simple algebras and cohomology groups of arbitrary fields*, Transactions of the American Mathematical Society **90** (1959), 73–112.

44. *On the semi-simplicity of group algebras*, Michigan Journal of Mathematics **6** (1959), 251–253.

45. *Some results on arithmetic functions*, Journal of the Mathematical Society of Japan **11** (1959), 275–290.

46. *On the convolution of arithmetic functions*, Riveon lematematica **13** (1959), 1–12 (in Hebrew; English summary).

47. *Finite dimensional central division algebras*, Proceedings of the American Mathematical Society **11** (1960), 28–31.

48. *Arithmetic linear transformations and abstract prime number theorems*, Canadian Journal of Mathematics **13** (1961), 83–109.

49. *Groups with representations of bounded degree. II*, Illinois Journal of Mathematics **5** (1961), 198–205.

50. *Derived functors in abelian categories*, Journal of Mathematical Mechanics **10** (1961), 971–994.

51. *Homology groups and double complexes for arbitrary fields*, Journal of the Mathematical Society of Japan **14** (1962), 1–25.

52. *On a lemma in elementary proofs of the prime number theorem*, Bulletin of the Research Council of Israel **10F** (1962), 101–108.

53. *Remarks on principal ideal rings*, Osaka Mathematical Journal **15** (1963), 59–69.

54. *Complexes of rings*, Israel Journal of Mathematics **2** (1964), 143–154.

55. *Generalized polynomial identities and pivotal monomials*, Transactions of the American Mathematical Society **114** (1965), 210–226.

56. *Identical relations in associative rings.* Some Aspects of Ring Theory C.I.M.E. Conference II. Circolo (1965), pp. 1–48.

57. *Jacobson-rings and Hilbert algebras with polynomial identities* (co-author C. Procesi), Annali di Mathematica Pura et Applicata **71** (1966), 61–72.

58. *Nil semi-groups of rings with a polynomial identity*, Nagoya Mathematical Journal **27** (1966), 103–111.

59. *Rational identities and applications to algebra and geometry*, Journal of Algebra **3** (1966), 304–359.

60. *Prime rings having polynomial identities with arbitrary coefficients*, Proceedings of the London Mathematical Society **17** (1967), 470–486.

61. *Rings with involution*, Israel Journal of Mathematics **6** (1968), 99–106.

62. *Correction to "Arithmetic linear transformations"*, Canadian Journal of Mathematics **21** (1969), 1–5.

63. *Identities in rings with involutions*, Israel Journal of Mathematics **7** (1969), 63–68.

64. *A noncommutative Hilbert basis theorem and subrings of matrices*, Transactions of the American Mathematical Society **149** (1970), 133–142.

65. *Rings of quotients and Morita contexts*, Journal of Algebra **17** (1971), 273–298.

66. *Embeddings in matrix rings*, Pacific Journal of Mathematics **36** (1971), 21–29.

67. *Some results on rings with polynomial identities*, Actes du Congres International des Mathematiciens, Vol. 1 (Nice, 1970), Gauthier-Villars, Paris, 1970, pp. 269–272.

68. *A note on PI-rings*, Israel Journal of Mathematics **10** (1971), 210–211.

69. *Nil radicals. Historical notes and some new results*, Colloq. Mathematical Society Janos Bolyai, Vol. 6: *Rings, Modules and Radicals* (Proc. Int. Colloq., Keszthely, Hungary), North-Holland, Amsterdam, 1971, pp. 47–65.

70. *On rings of quotients*, Symposia Mathematica **8** (1972), 149–164.

71. *On central division algebras*, Israel Journal of Mathematics **12** (1972), 408–420.

72. *On universal embeddings in matrix rings*, Journal of the Mathematical Society of Japan **25** (1973), 322–328.

73. *Polynomial identities and Azumaya algebras*, Journal of Algebra **27** (1973), 117–125.

74. *On supernilpotent radicals*, in *Estudos de Matematica*, Instituto de Alto Cultura, Lisbon, 1974, pp. 51–56.

75. *Jacob Levitzki, 1904–1956*, Israel Journal of Mathematics **19** (1974), 1–2.

76. *The generic division rings*, Israel Journal of Mathematics **17** (1974), 241–247.
77. *Polynomial identities*, Israel Journal of Mathematics **19** (1974), 183–199.
78. *Central embeddings in semi-simple rings*, Pacific Journal of Mathematics **56** (1975), 1–6.
79. *Identities and linear dependence*, Israel Journal of Mathematics **22** (1975), 127–137.
80. *On a central identity for matrix rings*, Journal of the London Mathematical Society **14** (1976), 1–16.
81. *Prime PI-rings* (co-author L.W. Small), Bulletin of the American Mathematical Society **83** (1977), 249–251.
82. *Alternating identities*, in *Ring Theory* (Proceedings of the Ohio University Conference on Ring Theory), Dekker, New York, 1978, pp. 1–14.
83. *Generic abelian crossed products and p-algebras* (co-author D. Saltman), Journal of Algebra **51** (1978), 76–87.
84. *Polynomials over division rings* (co-author L.W. Small), Israel Journal of Mathematics **31** (1978), 353–358.
85. *The polynomial identities of associative rings. Noetherian rings and rings with polynomials identities.* Proceedings of a conference, Durham, July (1978), pp. 1–38.
86. *Division algebras of degree 4 and 8 with involution* (co-authors L.H. Rowen and J.-P. Tignol), Bulletin of the American Mathematical Society **1** (1979), 691–693.
87. *Division algebras of degree 4 and 8 with involution* (co-authors L.H. Rowen and J.-P. Tignol), Israel Journal of Mathematics **33** (1979), 133–148.
88. *On the characteristic polynomial of a sum of matrices*, Linear and Multilinear Algebra **8** (1979/80), 177–182.
89. *Prime ideals in PI-rings* (co-author L. W. Small), Journal of Algebra **62** (1980), 358–383.
90. *Generic splitting fields*, in *Brauer Groups in Ring Theory and Algebraic Geometry* (Wilrijk, 1981), Lecture Notes in Mathematics **917**, Springer-Verlag, Berlin, 1982, pp. 1–24.
91. *Extension of derivations to central simple algebras*, Communications in Algebra **10** (1982), 797–803.
92. *Division algebras. A survey*, Contemporary Mathematics **13** (1982), 3–26.

93. *PI-algebras and their cocharacters* (co-author A. Regev), Journal of Algebra **78** (1982), 248–254.

94. *The sequence of codimensions of PI-algebras*, Israel Journal of Mathematics **47** (1984), 1–22.

95. *Kummer subfields of Malcev–Neumann division algebras* (co-author J.-P. Tignol), Israel Journal of Mathematics **50** (1985), 114–144.

96. *Symplectic modules* (co-author J.-P. Tignol), Israel Journal of Mathematics **54** (1986), 266–290.

97. *Totally ramified splitting fields of central simple algebras over Henselian fields* (co-author J.-P. Tignol), Institut Mathematique de l'Université Catholique de Louvain-La Neuve Vol. **36** (1983), iv1–iv9; Journal of Algebra **98** (1986), 95–101.

98. *An example in central division rings*, in *Perspectives in Ring Theory* (Antwerp, 1987), NATO Adv. Sci. Inst. Ser. C, Vol. 233, Kluwer, Dordrecht, 1988, pp. 85–92.

99. *Finite-dimensional representations of PI algebras* (co-author L.W. Small), Journal of Algebra **133** (1990), 244–248.

100. *GK-dimensions of corners and ideals* (co-author L.W. Small), Israel Journal of Mathematics **69** (1990), 152–160.

101. *Generic structures and division algebras*, Seminaire Mathématique Rapport 171, Univ. Catholique de Louvain, 1990, pp. 1–12.

102. *Galois splitting fields of a universal division algebra*, Journal of Algebra **143** (1991), 236–245.

103. *Highlights in the history of finite-dimensional central division algebras*, in *Ring Theory 1989*, Israel Mathematics Conference Proceedings, Vol. 1, The Weizmann Science Press of Israel, Jerusalem, 1989, pp. 404–430.

104. *Contributions of PI theory to Azumaya algebras*, in *Rings, Extensions, and Cohomology*, Lecture Notes in Pure and Applied Mathematics **159**, Dekker, New York, 1994, pp. 9–17.

105. *Elements of reduced trace* 0 (co-author L.H. Rowen), Israel Journal of Mathematics **87** (1994), 161–179.

106. *Affine algebras with polynomial identities* (co-author L.W. Small), Rendiconti del Circolo Matematico di Palermo (2) Supplement **31** (1993), 9–43.

107. *Finite-dimensional representation of PI algebras.II* (co-author L.W. Small),

Journal of Algebra **174** (1995), 848–855.

108. *Recognition of matrix rings II* (co-authors G. Agnarsson and J.C. Robson), Israel Journal of Mathematics **96** (1996), 1–13 (this volume).
109. *Finite-dimensional subalgebras of division rings* (co-author D. Baum), Israel Journal of Mathematics **96** (1996), 15–22 (this volume).
110. *Algebras over infinite fields, revisited* (co-author L.W. Small), Israel Journal of Mathematics **96** (1996), 23–25 (this volume).